EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,B

(A) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$= \frac{a^2+b^2-c^2+a^2+c^2-b^2+b^2+c^2-a^2}{abc}$

$= \frac{a^2+b^2+c^2}{2abc}$

(B) $\frac{\sin A}{a} + \frac{\sin B}{b} + \frac{\sin C}{c} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R} = \frac{3}{2R}$

(C) $\frac{\sin 2A}{a^2} = \frac{\sin 2B}{b^2} = \frac{\sin 2C}{c^2}$

$\Rightarrow \frac{2\cos A}{2Ra} = \frac{2\cos B}{2Rb} = \frac{2\cos C}{2Rc}$

$\Rightarrow \frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$\Rightarrow$ ΔABC equilateral

Sol.2 B,D

$r_1 = 2r_2 = 3r_3$

let $r_1 = k$, $r_2 = \frac{k}{2}$, $r_3 = \frac{k}{3}$

$s - a = \frac{\Delta}{k}$, $s - b = \frac{2\Delta}{k}$, $s - c = \frac{3\Delta}{k} \Rightarrow s = \frac{6\Delta}{k}$

$\Rightarrow a = \frac{5\Delta}{k}$, $b = \frac{4\Delta}{k}$, $c = \frac{3\Delta}{k}$

$\Rightarrow \frac{a}{b} = \frac{5}{4}$ & $\frac{a}{c} = \frac{5}{3}$

Sol.3 A,B,C,D

(A) $\frac{\Delta}{s-b} + \frac{\Delta}{s-c} = \frac{\Delta}{s-a} - \frac{\Delta}{s}$

$\Rightarrow \frac{s-b+s-c}{(s-b)(s-c)} = \frac{s-s+a}{s(s-a)}$

$\Rightarrow \frac{a}{(s-b)(s-c)} = \frac{a}{s(s-a)}$

$\Rightarrow \left(\frac{a-b+c}{2}\right) . \left(\frac{a+b-c}{2}\right)$

$= \left(\frac{a+b+c}{2}\right) . \left(\frac{b+c-a}{2}\right)$

$\Rightarrow a^2 - (b - c)^2 = (b + c)^2 - a^2$

$\Rightarrow 2a^2 = 2b^2 + 2c^2 \Rightarrow a^2 = b^2 + c^2 \Rightarrow \angle A = 90^\circ$

(B) $4R^2 (\sin^2 A + \sin^2 B + \sin^2 C) = 8R^2$

$\Rightarrow \left(\frac{1}{2} - \frac{1}{2}\cos 2A\right) + \left(\frac{1}{2} - \frac{1}{2}\cos 2B\right) + \left(\frac{1}{2} - \frac{1}{2}\cos 2C\right) = 2$

$\Rightarrow -\frac{1}{2} - \frac{1}{2} [\Sigma \cos 2A] = 0$

$\Rightarrow \Sigma \cos 2A = -1$

$\Rightarrow -1 - 4 \Pi \cos A = -1$

$\Rightarrow \Pi \cos A = 0 \Rightarrow$ A or B or C = 90°

(C) $r_1 = s \Rightarrow \frac{\Delta}{s-a} = s \Rightarrow \Delta = s(s - a)$

$\Rightarrow \frac{\Delta}{s(s-a)} = 1 \Rightarrow \tan \frac{A}{2} = 1$

$\Rightarrow \frac{A}{2} = \frac{\pi}{4} \Rightarrow A = 90^\circ$

(D) $2R = r_1 - r$

$\Rightarrow 2R = 4R \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

$- 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

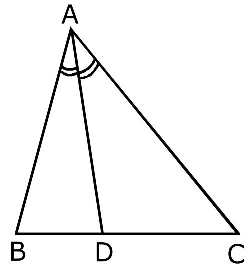
$\Rightarrow \frac{1}{2} = \sin \frac{A}{2} \cos \left(\frac{B+C}{2}\right)$

$\Rightarrow \sin^2 \frac{A}{2} = \left(\frac{1}{2}\right)^2 \Rightarrow \frac{A}{2} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\Rightarrow A = \frac{\pi}{2} \quad \therefore A \neq \frac{3\pi}{2}$

Sol.4 A,C,D

(A) $AD = \frac{2bc}{b+c} \cos \frac{A}{2}$



(C) $AD = \frac{bc}{b+c}\ 2\sqrt{\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{(s-b)(s-c)}}$

$= \frac{bc}{b+c}\ \frac{2.\Delta}{bc}\sqrt{\frac{bc}{(s-b)(s-c)}}$

$= \frac{1}{b+c} \times \frac{abc}{2R} \text{ cosec } \frac{A}{2} = \frac{abc \operatorname{cosec} \frac{A}{2}}{2R(b+c)}$

(D) $\frac{2\Delta}{(b+c)} \text{ cosec } \frac{A}{2}$

Sol.5 A,B

cosA cos B + sin A sin B sin C = 1

If sin C = 1 $\Rightarrow$ $\angle C = 90^o$

cos (A – B) = 1 $\Rightarrow$ A – B = 0 $\Rightarrow$ A = B

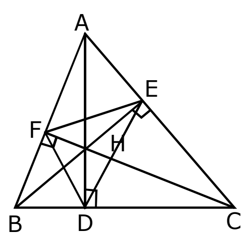
ΔABC is right angled isosceles triangle

Sol.6 A,B,C,D

$\angle E = \pi - 2B$

FE = R sin 2A

DE = R sin 2C



$\Delta = \frac{1}{2} R^2 \sin 2A \sin 2B \sin 2C$

(A) Perimeter

$\Delta DEF = R(\sin 2A + \sin 2B + \sin 2C)$

$= 4R \sin A \sin B \sin C$

$= 4R \frac{a.b.c}{2R.2R.2R} = \frac{a.bc}{2R^2}$

$\frac{\text{perimeter } \Delta DEF}{\text{perimeter } \Delta ABC} = \frac{abc}{2R^2(a+b+c)}$

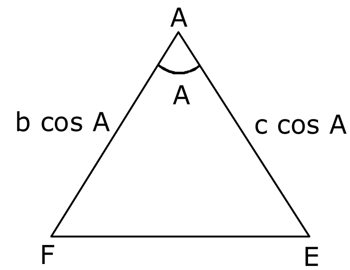
$= \frac{4R\Delta}{4R^2 s} = \frac{\Delta}{Rs} = \frac{r}{R}$

(B) $\Delta DEF = \frac{a'b'c'}{4R_{DEF}} = \frac{a \cos A b \cos B c \cos C}{4\left(\frac{R}{2}\right)}$

$\left\{\because R_{DEF} = \frac{R}{2}\right.$

$= \frac{2(abc)}{(4R)} \cos A \cos B \cos C = 2\Delta\pi \cos A$

(C) $\Delta AEF = \frac{1}{2} bc \cos^2 A \sin A = \Delta \cos^2 A$

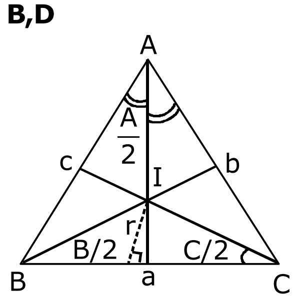


(D) $\frac{a'}{\sin(\pi - 2A)} = 2R_{DEF}$

$\Rightarrow \frac{a \cos A}{2 \sin A \cos A} = 2\ R_{DEF}$

$\Rightarrow \frac{2R}{2} = 2\ R_{DEF} \Rightarrow R_{DEF} = \frac{R}{2}$

Sol.7 B,D



$BI = r \text{ cosec } \frac{B}{2}, AI = r \text{ cosec } \frac{A}{2}$

$CI = r \text{ cosec } \frac{C}{2}$

$AI . BI . CI = \frac{r^3}{\sin\frac{A}{2} \sin\frac{B}{2} . \sin\frac{C}{2}}$

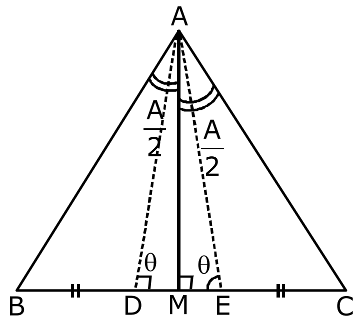
$= \frac{r^3}{r} 4R = 4Rr^2 \Rightarrow (B)$

$= \frac{abc}{\Delta} . \frac{\Delta^2}{s^2} = \frac{abc.r}{s} \Rightarrow (D)$

Sol.8 A,C,D

$\Delta ABD \cong \Delta ACE \Rightarrow \angle B = \angle C \Rightarrow (D)$

$BD = DE = EC = x$



$$\tan\theta = \frac{AM}{\frac{x}{2}} = \frac{2(AM)}{x}$$

$$\tan B = \frac{AM}{\frac{3x}{2}} = \frac{2(AM)}{3x}$$

$\Rightarrow \quad \tan\theta = 3 \tan B \Rightarrow (A)$

$$\tan\frac{A}{2} = \frac{3x}{2(AM)} \quad \{\because\ AM = \frac{x}{2}\tan\theta$$

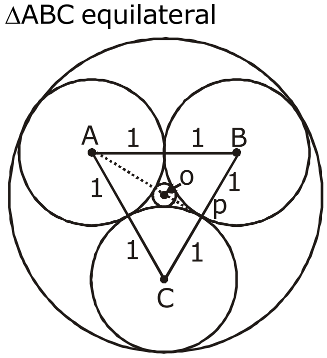
$$\tan\frac{A}{2} = \frac{3x}{x\tan\theta} \Rightarrow \tan\frac{A}{2} = \frac{3}{\tan\theta}$$

$$\therefore \quad \tan A = \frac{2\left(\frac{3}{\tan\theta}\right)}{1-\left(\frac{3}{\tan\theta}\right)^2}$$

$$\Rightarrow \quad \tan A = \frac{6\tan\theta}{\tan^2\theta - 9} \Rightarrow (C)$$

Sol.9 A,C

ΔABC equilateral



In ΔAPC, $\frac{AP}{1} = \tan 60^o \Rightarrow AP = \sqrt{3}$

$$AO = \frac{2}{3}\sqrt{3} = \frac{2}{\sqrt{3}}$$

radius of outer circle

$$= AO + 1 = \frac{2}{\sqrt{3}} + 1 = \frac{2+\sqrt{3}}{\sqrt{3}}$$

radius of smaller circle $= \frac{2}{\sqrt{3}} - 1 = \frac{2-\sqrt{3}}{\sqrt{3}}$

Sol.10 A,C,D

$(r_1 - r)(r_2 - r)(r_3 - r)$

Now

$$(r_1 - r) = 4R\sin\frac{A}{2}\left[\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right]$$

$$= 4R\sin\frac{A}{2}\cos\left(\frac{B+C}{2}\right)$$

$$\Rightarrow (r_1 - r) = 4R\sin^2\frac{A}{2}$$

IIIy $(r_2 - r) = 4R\sin^2\frac{B}{2}$ & $(r_3 - r) = 4R\sin^2\frac{C}{2}$

$$\pi(r_1 - r) = 64R^3\sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}$$

$$= 4R\left(4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)^2 = 4Rr^2 \Rightarrow (D)$$

$$= \frac{abc}{\Delta}\frac{\Delta^2}{s^2} = \frac{abc\Delta}{s^2} = \frac{(abc)^2}{4Rs^2}$$

$$= \frac{(abc)^2}{R(a+b+c)^2} \Rightarrow (C)$$

$$\Downarrow$$

$$= \frac{acb\Delta^2}{\Delta s^2}\cdot\frac{\Delta}{\Delta} = \frac{(abc)\Delta^3}{s^2\Delta^2} = \frac{abc\Delta^3}{s^2 s(s-a)(s-b)(s-c)}$$

$$= (abc)\left(\frac{\Delta}{s(s-a)}\right)\left(\frac{\Delta}{s(s-b)}\right)\left(\frac{\Delta}{s(s-c)}\right)$$

$$= abc\ \pi\tan\frac{A}{2} \Rightarrow (A)$$